32. $\lim\limits_{x \to -\infty} \dfrac{1}{\sqrt{x^2 + 2x} - x}$

33. What are the horizontal asymptotes of $y = \dfrac{1}{\sqrt{x^2 - 2x} - x}$? What are its vertical asymptotes?

34. What are the horizontal and vertical asymptotes of

$$y = \frac{2x - 5}{|3x + 2|}?$$

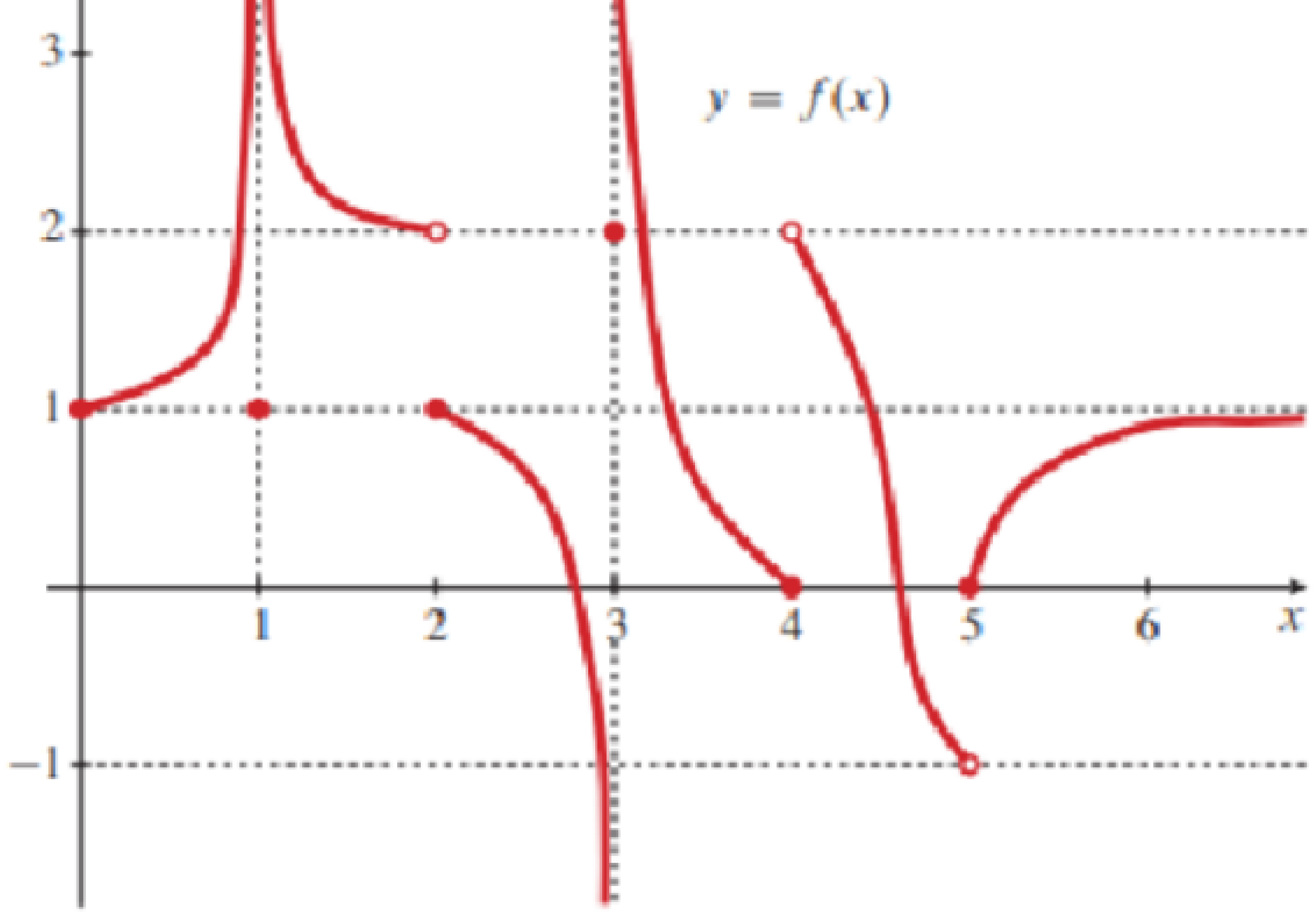

Figure 1.19

The function f whose graph is shown in Figure 1.19 has domain $[0, \infty)$. Find the limits of f indicated in Exercises 35–45.

35. $\lim_{x \to 0+} f(x)$

36. $\lim_{x \to 1} f(x)$

37. $\lim_{x \to 2+} f(x)$

38. $\lim_{x \to 2-} f(x)$

39. $\lim_{x \to 3-} f(x)$

40. $\lim_{x \to 3+} f(x)$

41. $\lim_{x \to 4+} f(x)$

42. $\lim_{x \to 4-} f(x)$

43. $\lim_{x \to 5-} f(x)$

44. $\lim_{x \to 5+} f(x)$

In Exercises 7–12, state where in its domain the given function is continuous, where it is left or right continuous, and where it is just discontinuous.

7. $f(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$

8. $f(x) = \begin{cases} x & \text{if } x < -1 \\ x^2 & \text{if } x \geq -1 \end{cases}$

76. (a) Sketch the curves $y = x^2$ and $y = x^4$ on the same graph. Where do they intersect?

(b) The function $f(x)$ satisfies:

$$\begin{cases} x^2 \le f(x) \le x^4 & \text{if } x < -1 \text{ or } x > 1 \\ x^4 \le f(x) \le x^2 & \text{if } -1 \le x \le 1 \end{cases}$$

Find (i) $\lim_{x \to -1} f(x)$, (ii) $\lim_{x \to 0} f(x)$, (iii) $\lim_{x \to 1} f(x)$.

Find the intervals on which the functions $f(x)$ in Exercises 25–28 are positive and negative.

25. $f(x) = \dfrac{x^2 - 1}{x}$

26. $f(x) = x^2 + 4x + 3$

26. $f(x) = x^2 + 4x + 3$

27. $f(x) = \dfrac{x^2 - 1}{x^2 - 4}$

28. $f(x) = \dfrac{x^2 + x - 2}{x^3}$

Using the Squeeze Theorem

74. If $\sqrt{5 - 2x^2} \leq f(x) \leq \sqrt{5 - x^2}$ for $-1 \leq x \leq 1$, find $\lim_{x \to 0} f(x)$.

75. If $2 - x^2 \leq g(x) \leq 2\cos x$ for all x, find $\lim_{x \to 0} g(x)$.

67. If $\lim\limits_{x \to 2} \dfrac{f(x) - 5}{x - 2} = 3$, find $\lim\limits_{x \to 2} f(x)$.

68. If $\lim\limits_{x \to 0} \dfrac{f(x)}{x^2} = -2$, find $\lim\limits_{x \to 0} f(x)$ and $\lim\limits_{x \to 0} \dfrac{f(x)}{x}$.

13. $\dfrac{x^2-4}{x-2}$ at $x=2$

14. $\dfrac{1+t^3}{1-t^2}$ at $t=-1$

15. $\dfrac{t^2 - 5t + 6}{t^2 - t - 6}$ at 3

16. $\dfrac{x^2 - 2}{x^4 - 4}$ at $\sqrt{2}$

17. Find k so that $f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ k - x^2 & \text{if } x > 2 \end{cases}$ is a continuous function.

18. Find m so that $g(x) = \begin{cases} x - m & \text{if } x < 3 \\ 1 - mx & \text{if } x \geq 3 \end{cases}$ is continuous for all x.

Evaluate the limits in Exercises 5–30 or explain why the
exist.

5. $\displaystyle\lim_{x\to 1}(x^2-4x+7)$

6. $\displaystyle\lim_{x\to 2}\frac{x^2}{1-x^2}$

7. $\displaystyle\lim_{x\to 1}\frac{x^2}{1-x^2}$

8. $\displaystyle\lim_{x\to 2}\frac{x^2-4}{x^2-5x+6}$

9. $\displaystyle\lim_{x\to 2}\frac{x^2-4}{x^2-4x+4}$

10. $\displaystyle\lim_{x\to 2-}\frac{x^2-4}{x^2-4x+4}$

11. $\lim_{x \to -2+} \frac{x^2 - 4}{x^2 + 4x + 4}$

12. $\lim_{x \to 4} \frac{2 - \sqrt{x}}{x - 4}$

13. $\lim\limits_{x\to 3} \dfrac{x^2 - 9}{\sqrt{x} - \sqrt{3}}$

14. $\lim\limits_{h\to 0} \dfrac{h}{\sqrt{x + 3h} - \sqrt{x}}$